A UNIVERSAL SQUARE-ROOT NYQUIST (M) FILTER DESIGN FOR DIGITAL COMMUNICATION SYSTEMS

Behrouz Farhang-Boroujeny (University of Utah: Salt Lake City, Utah, USA; farhang@ece.utah.edu)

Abstract— **Designing transmit and receive filters that are matched together and their combination satisfy the Nyquist condition is a classical problem in digital communication systems. In this paper, we propose a novel method for designing such filters. The proposed method is based on a universal cost function whose minimization leads to designs that can strike a balance between the stopband attenuation, the residual intersymbol interference (ISI), robust sensitivity to timing jitter and/or reduced peak-to-average power ratio (PAR). An iterative algorithm for finding the global minimum of the proposed cost function is suggested and its excellent performance is shown by presenting variety of design examples.**

Index Terms – **Nyquist filters, Filter design.**

1. INTRODUCTION

A classical problem in data communication is to design a pair of matched transmit and receive filters whose cascade is a Nyquist pulse-shape. Mathematically, this problem is phrased as follows. We wish to design a filter $H(z)$ such that $G(z) = H(z)H(z^{-1})$ satisfies the Nyquist criterion

$$\sum_{k=0}^{M-1} G\left(ze^{-j2\pi kf}\right) = M \tag{1}$$

where M is an integer called over sampling factor. It indicates the number of filter coefficients per symbol interval. Equation (1) expresses the Nyquist criterion in the frequency domain. In the time domain, the Nyquist criterion finds the form

$$g(n) = \begin{cases} 1, & n = 0 \\ 0, & n = mM,\ m \neq 0 \\ \text{arbitrary}, & n \neq mM \end{cases} \tag{2}$$

where $g(n)$ is the inverse z-transform of $G(z)$. Also, for our further reference later, we note that $g(n) = h(n) \star h(-n)$, where $h(n)$ is the inverse z-transform of $H(z)$ and $\star$ denotes convolution.

A filter $G(z)$ that satisfies (1) is called Nyquist (M), [1], [2]. Moreover, since when $|z| = 1$, $G(z) = H(z)H(z^{-1}) = |H(z)|^2$ and thus $H(z) = \sqrt{G(z)}$, we refer to $H(z)$ as square-root Nyquist (M) filter.

A design that limits $H(z)$ to satisfy the Nyquist conditions (1) and (2) exactly is generally too restrictive and thus may not lead to a satisfactory filter. There are other aspects in a real-world design that one may wish to consider and a design that strikes a good balance between these aspects is often more acceptable.

The various aspects that may be considered while designing $H(z)$ are:
1. The length of $H(z)$ should be minimized to reduce the implementation cost.
2. The Nyquist criterion set by (1) or (2) need not be satisfied exactly.
3. The transmission bandwidth and the stopband attenuation of $H(z)$ are often dictated by a frequency mask. $H(z)$ must fit within the mask.
4. To provide immunity against timing jitter, the magnitude of side-lobes of the impulse response $g(n) = h(n) \star h(-n)$ should be reduced.
5. To reduce the peak-to-average power ratio (PAR) of the modulated signal, one should design a square-root pulse-shape $h(n)$ with a reduced tail size.
Clearly, there are conflicting requirements among these, and one must give due consideration to the underlying tradeoffs during the design. This is what makes the design of Nyquist filters a challenging task, perhaps, compared with the conventional filter design. Several techniques exist in the literature for the design of digital Nyquist and/or digital matched filters whose cascade is a Nyquist filter [3]-[7]. However, most of these techniques are limited in considering or do not consider at all one more of the above aspects.

The goal of this paper is to give a novel formulation of the design of square-root Nyquist (M) filters that takes into account all the above issues and allow the designer to trade among the different aspects. By adopting a soft constraint approach and assigning a selectable weight to each constraint, the designer is given the freedom of tightening or loosening each constraint.

2. PROBLEM FORMULATION

The problem of designing a square-root Nyquist (M) filter

$$H(z) = \sum_{n=0}^{N} h(n)z^{-n} \tag{3}$$

may be formulated as follows. Let $\mathbf{h} = [h(0)\ h(1)\ \cdots\ h(N)]^{\mathrm{T}}$ and $\mathbf{e}(z) = [1\ z^{-1}\ \cdots\ z^{-N}]^{\mathrm{T}}$, where the superscript T denotes transposition, and note that (3) may be written as

$$H(z) = \mathbf{h}^{\mathrm{T}}\mathbf{e}(z) \tag{4}$$

Using (4) and recalling that $G(z) = H(z)H(z^{-1})$, we get

$$\begin{aligned} G(z) &= \left(\mathbf{h}^{\mathrm{T}}\mathbf{e}(z)\right)\left(\mathbf{h}^{\mathrm{T}}\mathbf{e}(z^{-1})\right) \\ &= \mathbf{h}^{\mathrm{T}}\mathbf{e}(z)\mathbf{e}^{\mathrm{T}}(z^{-1})\mathbf{h} \\ &= \mathbf{h}^{\mathrm{T}}\mathbf{R}(z)\mathbf{h}, \end{aligned} \tag{5}$$

where

$$\mathbf{R}(z) = \mathbf{e}(z)\mathbf{e}^{\mathrm{T}}(z^{-1}) = \sum_{n=-N}^{N} z^{-n}\mathbf{S}_n \tag{6}$$

and $\mathbf{S}_n$ are constant matrices whose elements are given by

$$[\mathbf{S}_n]_{k,l} = \begin{cases} 1, & k-l=n \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

Substituting (6) in (5), we obtain

$$G(z) = \sum_{n=-N}^{N} \left(\mathbf{h}^{\mathrm{T}}\mathbf{S}_n\mathbf{h}\right) z^{-n}. \tag{8}$$

For $G(z)$ to be a Nyquist (M) filter, $\mathbf{h}$ has to be chosen such that

$$\mathbf{h}^T\mathbf{S}_n\mathbf{h} = \begin{cases} 1, & n=0 \\ 0, & n=mM,\ m\neq 0. \end{cases} \tag{9}$$

These are a set of constraints that must be imposed while optimizing the coefficients of $H(z)$.

On the other hand, we note that $H(z)$ is a lowpass filter and as part of the design goal the magnitude response of $H(z)$ over its stopband has to be minimized. Following the notation of the (square-root) raised-cosine filters [8], assuming a rolloff factor α, and recalling that $H(z)$ is to be designed for a sampling rate M times faster than the symbol rate, we find that the stopband of $H(z)$ starts at the normalized frequency $f_o = \frac{1+\alpha}{2M}$ and ends at $1-f_o$. Noting this, we define the cost function

$$\xi_s = \int_{f_o}^{1-f_o} |H(e^{j2\pi f})|^2 df \tag{10}$$

and as part of the design we seek an $H(z)$ that results in a small ξ_s. Moreover, recalling that according to the Parseval's relation $\mathbf{h}^{\mathrm{T}}\mathbf{h} = \int_0^1 |H(e^{j2\pi f})|^2 df$ and by using (4) one will obtain $|H(e^{j2\pi f})|^2 = \mathbf{h}^{\mathrm{T}}\mathbf{e}(e^{j2\pi f})\mathbf{e}^{\mathrm{T}}(e^{-j2\pi f})\mathbf{h}$, (10) may be rearranged as

$$\xi_s = \mathbf{h}^{\mathrm{T}}\mathbf{h} - \int_{-f_o}^{f_o} |H(e^{j2\pi f})|^2 df = \mathbf{h}^{\mathrm{T}}\mathbf{\Phi}\mathbf{h} \tag{11}$$

where

$$\mathbf{\Phi} = \mathbf{I} - \int_{-f_o}^{f_o} \mathbf{e}(e^{j2\pi f})\mathbf{e}^{\mathrm{T}}(e^{-j2\pi f})df. \tag{12}$$

Performing the relevant integrals, the elements of $\mathbf{\Phi}$ are obtained as

$$\phi_{kl} = \begin{cases} 1-2f_o, & k=l \\ -2f_o\mathrm{sinc}(2f_o(k-l)), & k\neq l. \end{cases} \tag{13}$$

To summarize, the design of a square-root Nyquist (M) filter is performed by minimizing the cost function ξ_s of (11), subject to the constraints (9).

3. DESIGN PROCEDURE

FIR filters are usually designed to have a linear phase response. When $H(z)$ is a lowpass filter, the linear phase translates to an even symmetry of the filter coefficients, i.e., $h(n) = h(N-n)$. For a given filter order, N, this constraint on the filter coefficients, naturally, comes at some loss in the filter performance. However, the symmetry of the filter coefficient can be used to reduce the computational complexity of the filter significantly. It turns out that in most cases, for a given filter specifications, a linear phase design leads a lower complexity than its non-linear phase counter part. Noting this, in the rest of this paper we limit our study and give all the derivations for the cases where $H(z)$ is a linear phase FIR filter. To include this symmetry in the design formulation, we define

$$\mathbf{h}' = \begin{bmatrix} h(0) & h(1) & \cdots & h((N-1)/2) \end{bmatrix}^T$$

when N is odd, and

$$\mathbf{h}' = \begin{bmatrix} h(0) & h(1) & \cdots & h(N/2) \end{bmatrix}^T$$

when N is even. The vector $\mathbf{h}$ thus may be written in terms of $\mathbf{h}'$ as

$$\mathbf{h} = \mathbf{E}\mathbf{h}' \tag{14}$$

where $\mathbf{E} = \begin{bmatrix} \mathbf{I} \\ \mathbf{J} \end{bmatrix}$, $\mathbf{I}$ is the identity matrix and $\mathbf{J}$, for N odd, is the antidiagonal matrix with the antidiagonal elements of 1 and, for N even, is obtained by removing the first row of the latter antidiagonal matrix. Using (14), (9) and (10) are, respectively, rearranged as

$$\mathbf{h}'^T\mathbf{S}'_n\mathbf{h}' = \begin{cases} 1, & n=0 \\ 0, & n=mM,\ m\neq 0. \end{cases} \tag{15}$$

and

$$\xi_s = \mathbf{h}'^{\mathrm{T}}\mathbf{\Phi}'\mathbf{h}', \tag{16}$$

where $\mathbf{S}'_n = \mathbf{E}^{\mathrm{T}}\mathbf{S}_n\mathbf{E}$ and $\mathbf{\Phi}' = \mathbf{E}^{\mathrm{T}}\mathbf{\Phi}\mathbf{E}$.

The equalities defined by (15) suggest a set of hard constraints which may be unnecessary in an actual design. By relaxing on these constraints, one will gain in reducing ξ_s, i.e., in improving the stopband attenuation. We also note that to improve on the robustness of the received signal to timing jitter, one may extend (15) to include the tails of $g(n)$. For this purpose, we replace (15) by the set *soft* equalities

$$\mathbf{h}'^T\mathbf{S}'_n\mathbf{h}' \approx d_n, \quad n = 0, 1, \cdots, N \tag{17}$$

where d_n are a set of desired/target values.

Next, to combine (16) and (17), we first note that the set of equations (17) may be combined together and written in the compact form

$$\mathbf{B}\mathbf{h}' \approx \mathbf{d} \tag{18}$$

where $\mathbf{d} = [d_0\ d_1\ \cdots\ d_N]^{\mathrm{T}}$, $\mathbf{S}' = \left[\mathbf{S}_0'^{\mathrm{T}}\ \mathbf{S}_1'^{\mathrm{T}}\ \cdots\ \mathbf{S}_N'^{\mathrm{T}}\right]^{\mathrm{T}}$, $\mathbf{B} = \left(\mathbf{I} \otimes \mathbf{h}'^{\mathrm{T}}\right)\mathbf{S}'$, $\otimes$ denotes the Kronecher product, and, here, $\mathbf{I}$ is the identity matrix of size $N+1$. We also apply the Cholesky factorization to expand $\mathbf{\Phi}'$ as $\mathbf{\Phi}' = \mathbf{C}^{\mathrm{T}}\mathbf{C}$, where $\mathbf{C}$ is an upper triangular matrix and use this to rearrange (16) as

$$\xi_s = (\mathbf{C}\mathbf{h}')^{\mathrm{T}}\mathbf{C}\mathbf{h}' = \|\mathbf{C}\mathbf{h}'\|^2. \tag{19}$$

From this, we argue, to minimize ξ_s, one may choose to minimize the length of the vector $\mathbf{C}\mathbf{h}'$. Accordingly, we may also say that as part of our design goal, we wish to find a vector $\mathbf{h}'$ which also satisfies the soft equation

$$\mathbf{C}\mathbf{h}' \approx \mathbf{0} \tag{20}$$

where $\mathbf{0}$ is a column vector with zero elements.

Combining (18) and (20), we get

$$\mathbf{D}\mathbf{h}' \approx \mathbf{u} \tag{21}$$

where $\mathbf{D} = \begin{bmatrix} \mathbf{B} \\ \mathbf{C} \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} \mathbf{d} \\ \mathbf{0} \end{bmatrix}$. The approximation (21) is an over-determined system of soft equations that we seek its solution for the unknown vector $\mathbf{h}'$. We also note that since some of the rows of $\mathbf{D}$ contain linear combination of the elements of $\mathbf{h}'$, (21) is quadratic in $\mathbf{h}'$.

To solve (21), we define the error vector

$$\mathbf{v} = \mathbf{\Gamma}(\mathbf{D}\mathbf{h}' - \mathbf{u}) \tag{22}$$

where $\mathbf{\Gamma}$ is a diagonal matrix whose diagonal elements are a set of weights to be given to the elements of the difference $\mathbf{D}\mathbf{h}' - \mathbf{u}$. Larger weights are assigned to those elements whose minimization should be emphasized. Zero weight is assigned to those elements that should be treated as *don't care.* The optimum value of $\mathbf{h}'$ is obtained by minimizing the cost function

$$\xi = \|\mathbf{v}\|^2. \tag{23}$$

We note that since $\mathbf{D}\mathbf{h}'$ is quadratic in $\mathbf{h}'$, ξ is is a fourth order function of $\mathbf{h}'$. Hence, (23) may be a multi-modal function and its global minimum can only be found iteratively, if a proper initial choice of $\mathbf{h}'$ (close to its global minimum) could be made. Next, we propose an algorithm that operates along this line.

The square-root raised-cosine pulse-shape is a good and readily available choice for $\mathbf{h}$. From this, we pick the corresponding elements to initialize $\mathbf{h}'$ and follow the algorithm listed in Table I to find its optimum value. In this algorithm, the steps listed under iterations are executed multiple times until $\mathbf{h}'$ converges. For the numerical results presented in the next section, a preset iteration number 20 is used. However, we note that the algorithm usually converges within less than 10 iterations. Experiments with this algorithm show that it always leads to good designs. A few of these designs are presented in the next section. It is also worth noting that the algorithm presented in Table I is similar to those that have been developed in [9] and [10] and successfully used in designing filter banks.

TABLE I

SQUARE-ROOT NYQUIST (M) FILTER DESIGN ALGORITHM.

Inputs
N: filter order
M: oversampling factor
α: rolloff factor
$\mathbf{\Gamma}$: diagonal matrix of weight factors.
Initialization
○ Construct $\mathbf{S}'$.
○ Apply Cholesky factorization to $\mathbf{\Phi}'$ to obtain $\mathbf{C}$.
○ Choose a desired/target vector $\mathbf{d}$ and form the vector $\mathbf{u}$, accordingly.
○ Construct the initial vector $\mathbf{h}_0'$ from the samples of a square-root raised-cosine pulse-shape with the rolloff factor α.
○ Let $i = 0$.
Iterations
○ $\mathbf{B} = \left(\mathbf{I} \otimes \mathbf{h}'^{\mathrm{T}}\right)\mathbf{S}'$
○ $\mathbf{D} = \begin{bmatrix} \mathbf{B} \\ \mathbf{C} \end{bmatrix}$
○ $\mathbf{h}' = \left(\mathbf{D}^{\mathrm{T}}\mathbf{\Gamma}^2\mathbf{D}\right)^{-1}\mathbf{D}^{\mathrm{T}}\mathbf{\Gamma}^2\mathbf{u}$
○ $\mathbf{h}_{i+1}' = (\mathbf{h}_i' + \mathbf{h}')/2$
○ Increment i
Final step
○ $\mathbf{h}' = \mathbf{h}_i'$
○ Construct $\mathbf{h}$ from $\mathbf{h}'$

4. NUMERICAL EXAMPLES

The MATLAB function 'srNyquistM.m', presented in Appendix A, is used to generate all the results of this section. This program has the following inputs:

- `N`: The filter length.
- `M`: The oversampling factor. It is set equal to 5 for all the results presented in this section.
- `alpha`: The rolloff factor, α
- `gmaZ`: The weight factor for the center coefficient and all the zero-crossing points in $g(n)$. It is defined below as γ.
- `gmaT`: The weight factor for the tails of $g(n)$. It is defined below as γ' and is used to improve the robustness of the receiver to timing phase error/jitter.
- `eta`: The weight factor for the tails of $h(n)$. It is defined below as η and is used to improve on the peak-to-average power ratio (PAR) of the transmit signal.

Also presented in Appendix A is the function 'sr_cos_p.m' that is used to generate the coefficients of a square-root raised-cosine filter. With these two functions, an interested reader can replicate all the results that are presented in the following subsections.

A. Minimizing ISI / maximizing stopband attenuation

Tables II and III present the results of a series of Nyquist (M) filters that we designed using the MATLAB function 'srNyquistM.m'. The results compare the designed filters with the truncated square-root raised-cosine filters of the same length.

Table II compares the stopband attenuation of the two designs according to the formula

$$\rho_{\mathrm{SB}} = 10 \log \frac{\int_{f_0}^{1-f_0} |H_{\mathrm{srNyq}}(e^{j2\pi f})|^2 df}{\int_{f_0}^{1-f_0} |H_{\mathrm{srrc}}(e^{j2\pi f})|^2 df} \tag{24}$$

where $H_{\mathrm{srNyq}}(e^{j2\pi f})$ and $H_{\mathrm{srrc}}(e^{j2\pi f})$ are the frequency responses of the square-root Nyquist (M) and square-root raised-cosine filters, respectively.

Table III presents the relative ISI level of the two designs when both are sampled optimally at the middle of the corresponding pulse-shapes. The relative ISI level is defined as

$$\rho_{\mathrm{ISI}} = 10 \log \frac{\sum\limits_{n=mM, m\neq 0} g_{\mathrm{srNyq}}^2(n)}{\sum\limits_{n=mM, m\neq 0} g_{\mathrm{srrc}}^2(n)} \tag{25}$$

where $g_{\mathrm{srNyq}}(n)$ and $g_{\mathrm{srrc}}(n)$ are the pulse-shapes resulting from the square-root Nyquist (M) and sqaure-root raised-cosine designs, respectively.

TABLE II

THE RELATIVE STOPBAND ATTENUATION, (24), OF THE SQUARE-ROOT NYQUIST (M) AND THE TRUNCATED SQUARE-ROOT RAISED-COSINE FILTERS.

		γ			
	N	0.5000	1.0000	2.0000	10.0000
$\alpha = 0.5$	20	1.6178	0.4063	0.0740	−0.1704
	30	9.3794	9.0566	8.9711	8.0121
	40	19.1303	19.0376	18.9644	12.5494
	50	30.1334	29.9167	28.0668	14.9912
	60	41.0508	36.3266	29.9738	24.0538
$\alpha = 0.25$	20	3.5810	−1.0794	−2.7703	−3.3811
	30	2.5102	−0.4043	−1.3489	-1.7130
	40	2.2428	0.4603	−0.0709	-0.3763
	50	3.9911	2.9134	2.6099	2.2819
	60	8.9934	8.3452	8.1682	7.6964

To obtain the results of Tables II and III the diagonal elements of $\mathbf{\Gamma}$ are selected as follows. Unit weights are assigned to the elements of $\mathbf{Ch}'$. These elements are related to and control the stopband attenuation of the filter. A weight factor γ is assigned to the elements of $\mathbf{Bh}'$ that correspond to the constraints (9). Zero weights are assigned to the rest of the elements of $\mathbf{Bh}'$. Accordingly, by increasing γ, one can make the constraints (9) tighter. This will be at the cost of some loss in the stopband attenuation. The results presented in Tables II and III show how the stopband attenuation and residual ISI can be traded against each other by varying the weight factor γ. To obtain the results of Tables II and III we have set γ' and η equal to zero.

TABLE III

THE RELATIVE ISI LEVEL, (25), IN THE SQUARE-ROOT NYQUIST (M) AND THE TRUNCATED SQUARE-ROOT RAISED-COSINE FILTERS.

		γ			
	N	0.5000	1.0000	2.0000	10.0000
$\alpha = 0.5$	20	−18.7242	−8.2342	0.2200	3.4130
	30	12.6339	20.4501	22.3180	23.6261
	40	2.9576	4.0373	4.2225	6.8532
	50	23.0144	23.2353	23.9055	33.4174
	60	9.5795	11.1440	15.1168	25.9084
$\alpha = 0.25$	20	−4.0740	3.3571	13.7849	30.0271
	30	−5.2105	3.7271	14.4272	25.5533
	40	−16.2490	−6.3214	3.5310	9.5329
	50	−0.1070	10.2161	17.8382	20.7652
	60	9.3846	19.1388	23.5555	24.9116

Next, to further explore the results of the Nyquist (M) designs, we pick one of the designs from Tables II and III and study their behavior in more detail. Let us consider the case where $\alpha = 0.5$, $N = 30$, and $\gamma = 2$. In this case, there is a moderate 8.97 dB improvement in the stopband attenuation and a significant 22.3 dB improvement in the residual ISI level. To show the impact of the reduced ISI, in Figs. 1(a) and (b), we have presented the received signal constellations when a 64-QAM sequence is passed through a pair of matched filters obtained from the two designs. That is, we have assumed an ideal (i.e., distortionless) channel and no additive noise. As observed, the residual ISI arising from the raised-cosine design results in spread of constellation points. On the other hand, the Nyquist (M) design results in constellation points with no noticeable spreading.

It appears that by adopting the proposed design strategy, one can gain both in the time and frequency domain. Is this true? We answer this question by exploring the impulse response of the system. Such impulse responses which are obtained by combining a pair of transmit and receive filters from each design are presented in Figs. 2. From these plots, we make the following observation. The tails of the system impulse response are larger in the case of square-root Nyquist (M). Larger tail in the impulse response results in a higher sensitivity to timing jitter [7]. This is the price paid for higher attenuation in the stopband and lower residual ISI.

B. Designs with robust behavior against timing jitter

To reduce the sensitivity of a communication system to timing jitter, one may choose to design a pulse-shape $h(n)$ that leads to a combined impulse response $g(n) = h(n) \star h(-n)$ with reduced tail sizes. This can be easily done within the design frame work that was developed

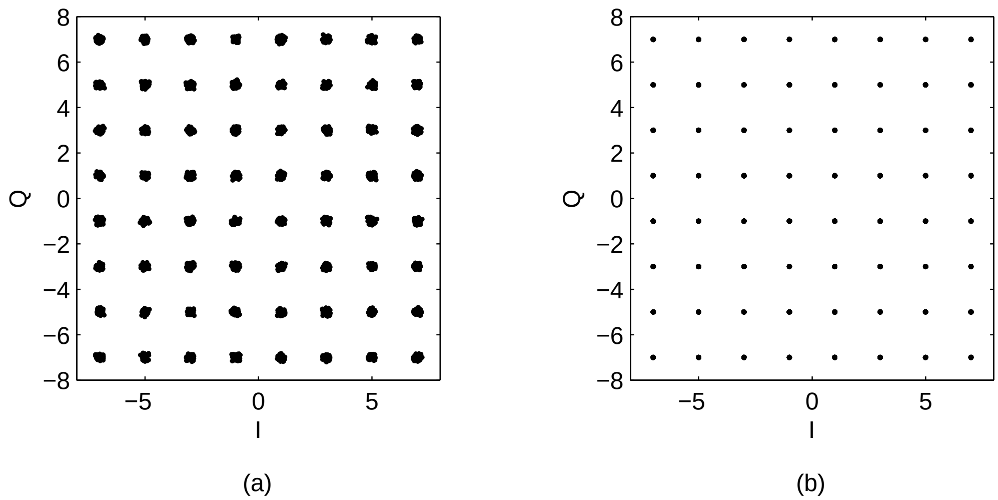

Fig. 1. Symbol constellation/eye pattern of a 64-QAM symbol sequence after passing through a pair of matched filters. (a) square-root raised-cosine filter. (b) square-root Nyquist (M) filter.

in this paper. In order to reduce the tail sizes, we simply assign some non-zero weights to the elements of $\mathbf{Bh}'$ that correspond to the tails of $g(n)$.

Fig. 3 presents signal constellations at the output of the matched filter at the receiver of one of our designs, for the timing phases of 0, 5, 10 and 15% of a symbol period. As expected, the proposed Nyquist (M) design is less sensitive to timing offset. For instance, at the timing offset of 15%, in the raised-cosine design the received symbols begin to overlap and errors can occur, even in the absence of channel noise. On the other hand, for the same timing offset, in the Nyquist (M) design, the constellation clusters remain separated. The cost for this robust behavior is a moderate loss in the stopband.

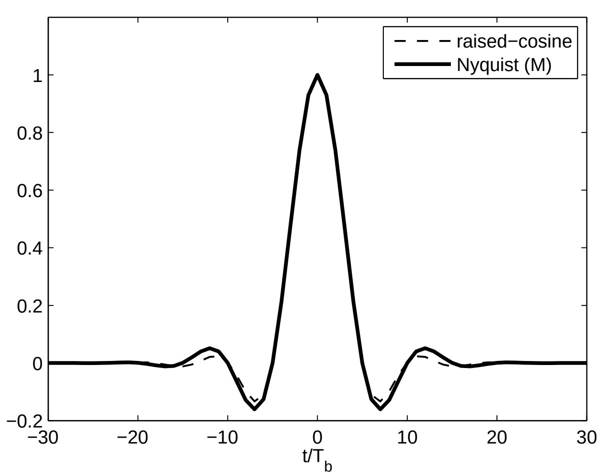

Fig. 2. Combined response of a pair of matched square-root raised-cosine and square-root Nyquist (M) filters.

The above design was arrived at through a few trials. In order to strike a balance between the reduced sensitivity to the timing offset and the stopband response of the design, we tried different choices of the weight factors. The design whose results are presented in Fig. 3 is obtained by using the following weight factors. A weight factor $\gamma = 5$ is assigned to the elements of $\mathbf{Bh}'$ that correspond to the zero crossing points and middle coefficient of the filter. A weight factor $\gamma' = 0.5$ is assigned to the rest of the tail samples of the response, i.e., to the elements of $g(n)$, for $|n| > M$ and $n \neq mM$, where m is an integer.

C. Designs with reduced PAR

The peak-to-average power ratio (PAR) is defined as the ratio of the peak signal power over the average power at the transmitter output. This is determined by the transmitter filter $h(n)$, and to reduce PAR, one may choose to reduce the size of the tails of $h(n)$. This can be easily included in the design formulation of this paper by adding an additional term $\eta \sum_{n \in \mathcal{T}_h} h^2(n)$ to the cost function ξ, where η is a weight factor and $\mathcal{T}_h$ is the set of n indices that correspond to the tails of $h(n)$. This design parameter is included and called `eta` in the MATLAB function 'srNyquistM.m'. Here, because of space limitation, we do not present any design of this type. However, an interested reader my try his own examples by assigning non-zero values to the parameter η.

5. CONCLUSION

We developed a generic cost function that could be used to design a wide range of transmit/receive filters in the application of digital communication systems. An iterative algorithm for minimization of the proposed cost function and a MALTAB function for its implementation were presented. A number of design examples that demonstrated the capabilities as well as the versatility of the proposed method were also presented.

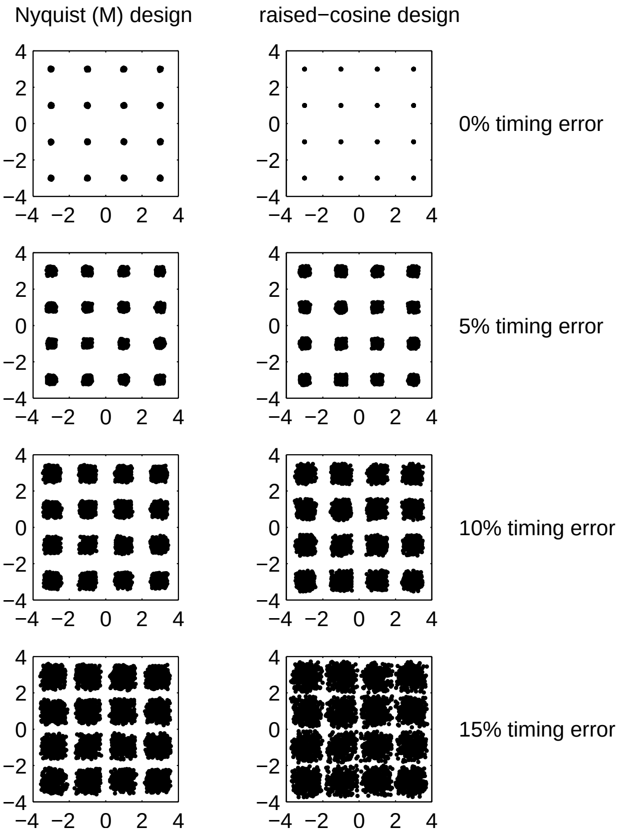

Fig. 3. Demonstration of the robust behavior of a Nyquist (M) filter to timing phase offset, through eye patterns. The eye patterns arising from a raised-cosine pulse are also presented.

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Appendix A: MTALAB Functions for Square-Root Nyquist (M) Filter Design

```
function h=srNyquistM(N,M,alpha,gmaZ,gmaT,eta,itns);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Square-root Nyquist (M) filter design                  %
% parameters:                                            %
% N: filter order (filter length = N+1)                  %
% M: number of samples per symbol period                 %
% alpha:  rolloff factor (range 0 to 1)                  %
% gmaZ: Weight factor for middle tap and zero crossings %
% gmaT: Weight factor for the tails of g=h*h             %
% eta:  Weight factor for tails of h                     %
% itns: No. of itns for the least-squares optimization  %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Set up the weight matrix Gamma %%%
Gamma=zeros(1,1+N);Gamma(M+2:end)=gmaT;Gamma(1:M:end)=gmaZ;
Gamma=[Gamma ones(1,1+N/2)];
Gamma2=Gamma.^2; Gamma2=diag(Gamma2);
%%% Initial filter %%%
h=sr_cos_p(N,M,alpha);
if rem(N+1,2)==0 h1=h(1:(N+1)/2);
          else h1=h(1:N/2+1); end
Lh1=length(h1);
%%% Set up constraint matrices, Sn %%%
S=zeros(N+1,N+1,N+1);
for n=1:N+1
    for k=1:N+1
       for l=1:N+1
           if (k-l)==n-1 S(k,l,n)=1; end
       end
    end
end
%%% Set up the matrix Phi %%%
Phi=zeros(N+1,N+1);
fo=(1/2/M)*(1+alpha);
for k=1:N+1
    for l=1:N+1
       if k==l Phi(k,l)=1-2*fo;
       else Phi(k,l)=-2*fo*sinc(2*fo*(k-l)); end
    end
end
Phi=Phi+1e-10*eye(size(Phi)); %to stabilize Cholosky
%%% Form the matrices Sn' and Phi' %%%
I=eye(Lh1); J=hankel([zeros(Lh1-1,1); 1]);
if rem(N+1,2)==1 J=J(2:end,:); end
E=[I; J]; Phi1=E'*Phi*E; S1=zeros(Lh1,Lh1,N+1);
for n=1:N+1  S1(:,:,n)=E'*S(:,:,n)*E;  end
%%% Add tail constraint to reduce PAR %%%
X=zeros(size(Phi1)); X=diag(X); X(1:end-M)=eta;
Phi1=Phi1+diag(X);
%%% Iterative lease-squares optimization %%%
C=chol(Phi1); % Choloskey factorization
for kk=1:itns
    B=[ ];   for n=1:N+1   B=[B; h1'*S1(:,:,n)];   end
    D=[B; C];  u=[1; zeros(N+Lh1,1)];
    h1=(h1+inv(D'*Gamma2*D)*(D'*Gamma2*u))/2;
end
h=E*h1;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Square-root raised-cosine filter %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function h=sr_cos_p(N,M,alpha)
t=[-N/2:1:N/2]'/M; h=zeros(size(t));
for k=1:length(t)
    if t(k)==0  h(k)=1-alpha+4*alpha/pi;
       elseif (t(k)==(-1/(4*alpha)))|(t(k)==(1/(4*alpha)))
          h(k)=(alpha/sqrt(2))*((1+2/pi)*sin(pi/4/alpha)+...
             (1-2/pi)*cos(pi/4/alpha));
       else  h(k)=(sin(pi*(1-alpha)*t(k))+...
             4*alpha*t(k)*cos(pi*(1+alpha)*t(k)))/...
                (pi*t(k)*(1-(4*alpha*t(k))^2));
    end
end
h=h/sqrt(M);
```