A LOW COMPLEXITY REAL-TIME MIMO-PREPROCESSING FOR FIXED-COMPLEXITY SPHERE DECODER

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ABSTRACT

Modern Multiple-Input Multiple-Output (MIMO) communication systems place huge demands on embedded processing resources in terms of throughput, latency and resource utilization. State-of-the-art MIMO detector algorithms, such as Fixed-Complexity Sphere Decoding (FSD), rely on efficient channel preprocessing involving numerous calculations of the pseudo-inverse of the channel matrix by QR Decomposition (QRD) and ordering. These highly complicated operations can quickly become the critical prerequisite for real-time MIMO detection, exaggerated as the number of antennas in a MIMO detector increases. This paper describes a sorted QR decomposition (SQRD) algorithm extended for FSD, which significantly reduces the complexity and latency of this preprocessing step and increases the throughput of MIMO detection. It merges the calculations of the QRD and ordering operations to avoid multiple iterations of QRD. Specifically, it shows that SQRD reduces the computational complexity by over 60-70% when compared to conventional MIMO preprocessing algorithms. In 4×4 to 7×7 MIMO cases, the approach suffers merely 0.16-0.2 dB reduction in Bit Error Rate (BER) performance.

1. INTRODUCTION

The continually increasing processing demands for higher data rates in modern wireless communication systems has led to the increased adoption of MIMO [1] in wireless standards such as IEEE 802.11n [2] and 802.16e, where multiple antennas are employed at both the transmitter and receiver ends of the communication channel. MIMO schemes achieve superior channel capacity, throughput and diversity over single antennas at the cost of increased computational complexity of detection algorithms in the receiver. This places huge demand on embedded processing resources in terms of throughput, latency and resource utilization, particularly when more antennas are deployed in modern MIMO systems.

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Among modern high performance MIMO detection algorithms, the decoding algorithms employed typically encounter problems with one or more of large computational complexity [3], reduced BER performance [4, 5], or non-deterministic data dependent behaviour [6, 7]. FSD [8, 9] overcomes these drawbacks, bringing deterministic structure and fixed complexity. The fundamental idea of FSD is to search, independently of the noise level, over only a fixed number of candidate in the searching sphere [8]. Hence, the decoder can offer deterministic behaviour, near ideal decoding performance and a highly parallel structure which is ideal for real-time implementation [8].

To achieve quasi-ML performance, FSD relies on a particular channel matrix ordering for preprocessing, where the channels and signals are ordered according to the distortion experienced, before undergoing upper triangularization. In original FSD, V-BLAST [4] is used to produce this channel order. However, the large computational complexity of this step ($\mathrm{O}(M_t^4)$) presents a complex prerequisite for real-time MIMO, particularly with the increasing number antennas in modern MIMO detectors.

A reduced complexity linear detection algorithm adapting a sorted QR decomposition (SQRD) of the channel matrix for ZF and MMSE criterion was presented in [10]. In this paper, we extend the SQRD algorithm to nonlinear MIMO detection algorithms, specifically FSD, to effect an order-of-magnitude reduction in preprocessing complexity whilst incurring only a minor degradation in BER performance. Specifically, it is shown how SQRD reduces the computational complexity of channel preprocessing by over 60% for 4×4 to 7×7 MIMO cases, whilst demonstrating very low BER degradation of 0.16-0.2 dB.

The remainder of the paper is organized as follows. Section 2 introduces related background about FSD and preprocessing. Section 3 describes SQRD for FSD preprocessing. Section 4 and 5 compare the performance and complexity of this SQRD and V-BLAST ordering for FSD.

2. BACKGROUND AND MOTIVATION

We consider a generic MIMO communication system with topology formulated as in (1).

$$\mathbf{y} = \mathbf{Hs} + \mathbf{n} \tag{1}$$

A transmitter sends data, **s**, of M_c-bit QAM symbol vector through M_t transmit antennas across $N_r \times M_t$ complex communication channel **H**, where it is corrupted by multipath distortion and white Gaussian noise **n** of variance σ_n^2, and the average transmit power of each antenna is normalized to one, i.e. $E\left\{\mathbf{ss}^H\right\} = \mathbf{I}$ and $E\left\{\mathbf{nn}^H\right\} = \sigma_n^2\mathbf{I}$. The received signal, **y**, is then sensed by N_r antennas at the receiver. Typically, the communications channel is used as a set of parallel flat fading subchannels (the fading channel gains are perfectly known by the receiver) via Orthogonal Frequency Division Multiplexing (OFDM) at the transmitter, with each subchannel decoded separately at the receiver.

Practical MIMO detectors employed in the receiver encounter problems resulting from either exhaustive search complexity [3], reduced complexity with reduced BER performance [4], huge complexity when quasi-ML performance is required [5], or reduced average complexity with non-deterministic data dependent behaviour [6, 7]. FSD is a notable exception, offering deterministic behaviour, quasi-ML decoding performance and a highly parallel structure which is ideal for real-time implementation [8]. It employs a three step tree decoding scheme, as illustrated in Fig. 1.

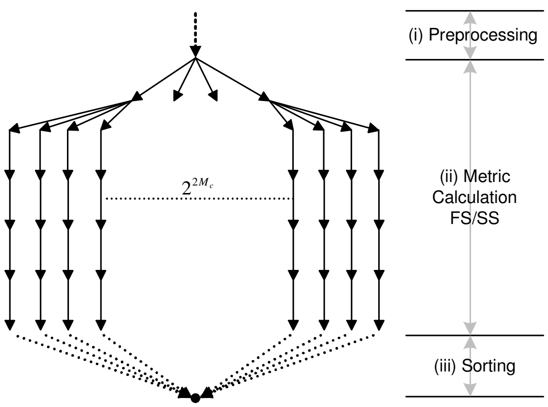

Fig. 1. FSD Tree Structure, M_t = 6, *nfs* = 2

(i) *Preprocessing* involves generating the channel order, an upper triangular version **R** of the channel matrix **H**, and initialising the centre of the FSD detection sphere using Zero Forcing (ZF) detection.

(ii) *Metric calculation* involves in two phases: a *Full Search* (FS) phase that fully enumerates the search space for the received symbols which have experienced the worst channel conditions, followed by a *Single Search* (SS) phase where only the single likeliest candidates are selected for the remaining layers [8]. For full diversity, the number of layers which should undergo FS and SS (*nfs* and *nss* respectively) are given by equations (2) and (3).

$$nfs = \lceil \sqrt{M_t} - 1 \rceil. \tag{2}$$

$$nss = M_t - nfs. \tag{3}$$

(iii) *Sorting* includes determining the most likely of the candidate detected symbols from each branch of the FSD tree by selecting that with the lowest Accumulated Partial Euclidean Distance (APED) value.

When using ZF criteria, the detected symbols $\tilde{\mathbf{s}}_{ZF}$ is defined using the pseudo inverse of the channel matrix, and σ_n^2 is the noise covariance. As such, the estimation errors of the different layers correspond to the main diagonal elements of the error covariance matrix Φ_{ZF} in (4) [10]. Small eigenvalues of the $\mathbf{H}^H\mathbf{H}$ in (4) will lead to large errors due to noise amplification.

$$\begin{aligned} \Phi_{ZF} &= E\left\{(\tilde{\mathbf{s}}_{ZF} - \mathbf{s})(\tilde{\mathbf{s}}_{ZF} - \mathbf{s})^H\right\} \\ &= \sigma_n^2(\mathbf{H}^H\mathbf{H})^{-1} = \sigma_n^2\mathbf{R}^{-1}\mathbf{R}^{-H} \end{aligned} \tag{4}$$

The key idea of FSD is to particularly order the channel matrix by the channel noise amplification in the preprocessing. The key function of the preprocessing is to determine the order of which received symbols undergo FS and which undergo SS. To do this, V-BLAST is employed to determine this order according to the noise amplification encountered on each channel as in (4) [10]. It determines the order from FS to SS stages, in the same direction as signal detection, experiencing matrix inversion, nulling, cancelling ($O(M_t^3)$ complexity) and ordering. Hence M_t iterations of this process leads to V-BLAST ordering experiencing $O(M_t^4)$ complexity. Practically, real-time manifestation of this step has not been achieved and lower complexity alternatives are desired.

A number of sub-optimal preprocessing approaches have been proposed for alternative MIMO detection schemes, e.g. [11, 12], but these are unsuitable for FSD since they order the channel matrix in ascending order of signal distortion, rather than the mixed descending/ascending FS/SS order required by FSD. In particular, SQRD has proven a highly attractive alternative to V-BLAST in other MIMO detection strategies as it offers very low complexity alternative, enabling quasi-ML performance whilst reducing computational complexity of pre-processing for MIMO detection by one order of magnitude. However since it does not apply the descending/ascending ordering required, SQRD is not directly applicable suitable to FSD. In Section 3 we present the first adaptation of SQRD for FSD-based MIMO detection.

3. SORTED QR DECOMPOSITION FOR FSD

SQRD achieves sub-optimal performance to V-BLAST, but with much less complexity [10]. The algorithm detects signals based on the robustness of the channels, generating a triangularised matrix. However, this is not suitable for FSD preprocessing, since the descending robustness order of SQRD is different from FSD's requirement which has different robustness orders in the FS and SS stages. However, a combination of SQRD and FSD ordering could offer the capability to jointly calculate the ordering and triangularisation, avoiding numerous matrix inversion iterations, reducing the preprocessing complexity by one order of magnitude.

Since in FSD, signals in FS layers should be detected ahead of SS layers (the opposite order to the channel matrix triangularisation by QRD). SQRD with FSD could be combined to determine the channel order from SS to FS layers, with the same direction as the channel matrix triangularisation by QRD. The channel matrix can be ordered based on the noise amplification, but in each of the i iterations of matrix triangularisation, the k=*min*(*nfs*+1,M_t-*i*+1) worst channel (e.g. k_i^{th}) is selected based on norm of $\mathbf{H}$ (e.g. diagonal element of $\mathbf{H}^H\mathbf{H}$), and permuted with the i^{th} channel before orthogonalization using Gram-Schmidt process, e.g., in the order of (2,3,4,1) for 4×4 MIMO, or (3,4,5,6,7,8,2,1) for 8×8 MIMO. Therefore, it allows preprocessing to combine the channel ordering and triangularisation with much less complexity.

A simple example of SQRD for 4×4 FSD MIMO is given in Fig. 2, where *nfs* $= 1$. $\mathbf{Q}$ is initialised by $\mathbf{H}$, $\mathbf{R}$ by zero, and **order** vector is sequentially initialised for the initial channel matrix order. The **norm** vector is obtained from the norm of $\mathbf{H}$. To determine the SS layer channels, in each iteration the channel with $(nfs+1)^{th}$ minimum norm (e.g. k_i^{th} channel) is permuted with the i^{th} channel. The same columns in $\mathbf{R}$ and same elements in **order** and **norm** are also permuted. In addition, $\mathbf{Q}$, $\mathbf{R}$ and **norm** are updated by Gram-Schmidt for orthogonalization [13]. In the next iteration, these processes are operated for the remaining channels, and so on so forth.

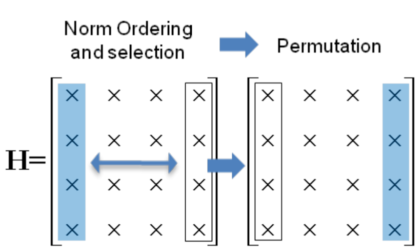

Fig. 2. Ordering Steps of SQRD for FSD

After all the SS layers have been determined, the FS layers are processed. In each iteration, the channel with $(M_t-i+1)^{th}$ minimum of the remaining norm is permuted with the i^{th} channel. Meanwhile, $\mathbf{R}$, **order** and **norm** are also permuted, and $\mathbf{Q}$, $\mathbf{R}$ and **norm** are updated by Gram-Schmidt process as well. Hence, $\mathbf{Q}$, $\mathbf{R}$ and **order** can be recursively obtained by starting from level $i=1$ and working forward until level $i=M_t$. As such, the required order can be obtained. The SQRD ordering can also be extended for higher dimensional cases, e.g., 5×5 to 7×7 MIMO.

Fig. 3 illustrates a diagram of the order decision and signal detection direction for the SQRD in 4×4 FSD MIMO. The channel order and **Q/R** matrices are jointly decided from SS to FS phase by checking from 1^{st} channel to the last (4^{th}), with the opposite direction of signal detection. At each channel iteration, channel selection, permutation, orthogonalisation, cancellation and ordering are experienced. The process works until the last layer in FS phase finish its decision.

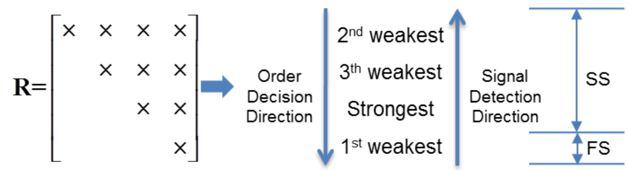

Fig. 3. Order Decision and Signal Detection Direction by SQRD Preprocessing

Algorithm 1 gives the pseudo code for the SQRD for FSD. In Phase 1, $\mathbf{Q}$ is initialised by $\mathbf{H}$, where $\mathbf{q}_i$ is the i^{th} column of $\mathbf{Q}$; $\mathbf{R}$ is initialised by $\mathbf{0}_{M_t}$; correspondingly, **order**, **norm** and *nfs* are also initialized (i.e., line 2-6). In Phase 2, the channel with k^{th} lowest norm is selected for every i^{th} iteration to permute with i^{th} channel. The relevant column in $\mathbf{R}$, along with the elements in **order** and **norm** are also permuted (i.e., line 11) before their orthogonalization (i.e., line 12-18) which is similar to the orthogonalization stage in [10]. As such, the unitary matrix $\mathbf{Q}$, upper-triangular matrix $\mathbf{R}$, and their **order** are obtained.

SQRD merges the calculations of ordering and triangularisation, and avoids the iterative matrix inversion operation inherently in V-BLAST. Hence, SQRD ordering reduces the M_t iterations of QRD to merely one, decreasing the computation complexity by one order of magnitude whilst maintaining the quasi-ML performance. In the next section, the effect of this complexity reduction on the decoding performance of an FSD-based detector is analysed.

4. PERFORMANCE EVALUATION

To compare the relative detection performance of SQRD and V-BLAST preprocessed detectors, we measure the BER performance of each scheme as a function of the SNR through Monte Carlo simulations for a variety of MIMO topologies (4×4 - 7×7) exploiting 16-QAM modulation. In each case the simulation consists of 50,000 channel realizations with 200 symbols transmitted in every channel realization. Fig. 4 illustrates BER performance of each case.

As Fig. 4 shows, SQRD-based ordering enables performance close to that of the V-BLAST solution. Specifically,

input : $\mathbf{H}$, M_t
output: $\mathbf{Q}$, $\mathbf{R}$, **order**

Phase 1: *Initialization*
$\mathbf{Q} = \mathbf{H}, \mathbf{R} = \mathbf{0}_{M_t}$,
$\mathbf{order} = [1, \cdots, M_t], nfs = \lceil \sqrt{M_t} - 1 \rceil$
for $i \leftarrow 1$ **to** M_t **do**
  $\mathbf{norm}_i = \|\mathbf{q}_i\|^2$
end

Phase 2: *SQRD ordering*
for $i \leftarrow 1$ **to** M_t **do**
  $k = min\,(nfs + 1, M_t - i + 1)$
  $k_i = \underset{j=i,\cdots,M_t}{\arg\min^{k}}\ \mathbf{norm}_j$
  Exchange columns i and k_i in $\mathbf{R}$, **order**, **norm** and $\mathbf{Q}$
  $r_{i,i} = \sqrt{\mathbf{norm}_i}$
  $\mathbf{q}_i = \mathbf{q}_i / r_{i,i}$
  for $l \leftarrow i + 1$ **to** M_t **do**
    $r_{i,l} = \mathbf{q}_i^H \cdot \mathbf{q}_l$
    $\mathbf{q}_l = \mathbf{q}_l - r_{i,l} \cdot \mathbf{q}_i$
    $\mathbf{norm}_l = \mathbf{norm}_l - r_{i,l}^2$
  end
end

Algorithm 1: Sorted QR decomposition for FSD

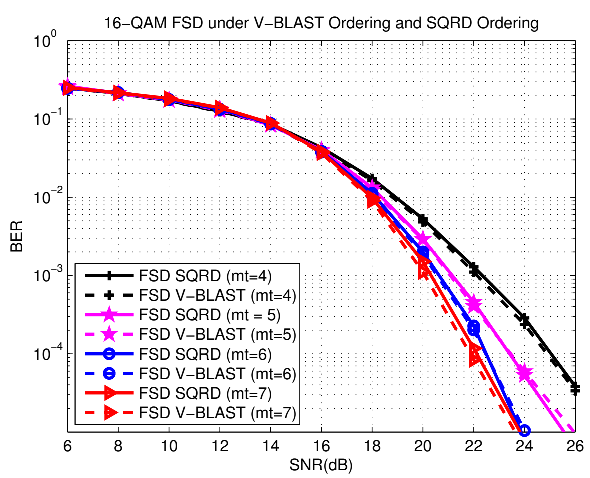

Fig. 4. SQRD/V-BLAST FSD BER Performance Comparison

for 4×4 - 6×6 MIMO, SQRD suffers only 0.16 dB performance degradation, rising to 0.2 dB degradation for 7×7 MIMO at BER $= 10^{-4}$. In light of this performance reduction, however mild, the viability or otherwise of SQRD as an alternative to V-BLAST depends on the complexity reduction achieved. This is calculated in Section 5.

5. COMPLEXITY EVALUATION

To provide an absolute measure of the complexity reduction, we have calculated the number of cycles required for V-BLAST and SQRD-based preprocessing. This involved accounting for each calculation in Algorithm 1, and assiging a weight to each different type of operation based on the number of cycles required by a modern DSP processor [14] to calculate each - 1 complex addition requires 2 cycles, 1 complex multiplication requires 6 cycles, scalar addition/multiplication require a single cycle each and finally division/square root required 6-7 cycles [14]. According to these metrics, the relative complexities of FSD preprocessing exploiting V-BLAST or SQRD are given in equations (5) and (6) respectively.

$$f_{FSD\_VBLAST} = \frac{7}{3}M_t^4 + 21M_t^3 + \frac{175}{6}M_t^2 + \frac{1}{2}M_t \quad (5)$$

$$f_{FSD\_SQRD} = \frac{28}{3}M_t^3 + \frac{45}{2}M_t^2 + \frac{1}{6}M_t - 4 \quad (6)$$

As these show, the complexity reduction affected by SQRD is significant. By reducing the number of QRD iterations from M_t (V-BLAST) to 1 (SQRD), the overall complexity of the preprocessing stpe has decreased by a one order of magnitude, respectively from O(M_t^4) to O(M_t^3). This reduction is corroborated by Fig. 5, which describes the number of arithmetic operations and required number of processing cycles for both VBLAST and SQRD for varying MIMO topologies.

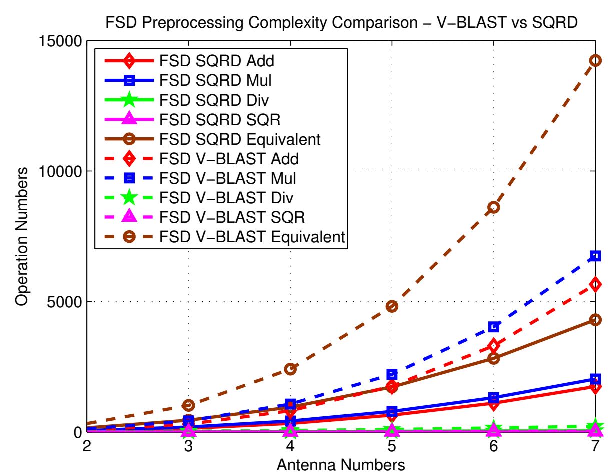

Fig. 5. FSD Preprocessing Complexity Comparison

To indicate how the absolute level of this complexity reduction varies with the topology of the MIMO system, consider Table. 1. As this shows, the number of operations is reduced by around 60-70% for 4×4 - 7×7 MIMO. The absolute level of saving increases with M_t.

Table 1. FSD Preprocessing Complexity Comparison

M_t	4	5	6	7
FSD-VBLAST	2410	4815	8613	14238
FSD-SQRD	954	1726	2823	4301
Saving Ratio (%)	60.4	64.2	67.2	69.8

In summary, whilst Section 4 revealed a mild reduction in performance as a consequence of employing SQRD as opposed to V-BLAST for FSD preprocessing, the resulting reduction in computational complexity is sufficient to justify this reduction.

6. CONCLUSION

In this paper the use of SQRD ordering for FSD preprocessing has been proposed. It has been show how this scheme enables an order-of-magnitude reduction in the computational complexity of the FSD preprocessing whilst quasi-ML detection performance is maintained and very low BER degradation experienced. Specifically, it has been shown how using SQRD avoids the multiple iterations of QRD inherent in the standard V-BLAST algorithm by jointly calculating the QRD and matrix ordering operations. This has been shown to enable a 60-70% reduction in computational complexity of the FSD preprocessing operation for 4×4 to 7×7 MIMO cases, while suffering only a 0.16-0.2 dB BER reduction at SNR of 10^{-4}.

7. REFERENCES

[1] Gerard J Foschini, "Layered Space-Time Architecture for Wireless Communication in a Fading Environment When Using Multi-Element Antennas," *Bell Labs Technical Journal*, vol. 1, no. 2, pp. 41–59, 1996.

[2] IEEE802.11n, "802.11n-2009 IEEE Local and metropolitan area networks–Specific requirements Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications Amendment 5: Enhancements for Higher Throughput," 2009.

[3] M O Damen, H El Gamal, and G Caire, "On Maximum-Likelihood Detection And The Search For The Closest Lattice Point," *Information Theory, IEEE Transactions on*, vol. 49, no. 10, pp. 2389–2402, 2003.

[4] P.W. Wolniansky, G.J. Foschini, G.D. Golden, and R.A. Valenzuela, "V-BLAST: An Architecture for Realizing Very High Data Rates Over The Rich-Scattering Wireless Channel," in *1998 URSI International Symposium on Signals, Systems, and Electronics. Conference Proceedings*, 1998, pp. 295–300.

[5] Zhan Guo and P Nilsson, "Algorithm and Implementation of the K-Best Sphere Decoding for MIMO Detection," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 3, pp. 491–503, Mar. 2006.

[6] E Viterbo and J Boutros, "A Universal Lattice Code Decoder For Fading Channels," *Information Theory, IEEE Transactions on*, vol. 45, no. 5, pp. 1639–1642, July 1999.

[7] A Burg, M Borgmann, M Wenk, M Zellweger, W Fichtner, and H Bolcskei, "VLSI Implementation of MIMO Detection Using The Sphere Decoding Algorithm," *IEEE Journal of Solid-State Circuits*, vol. 40, no. 7, pp. 1566–1577, July 2005.

[8] Luis Barbero and John Thompson, "Fixing the Complexity of the Sphere Decoder for MIMO Detection," *IEEE Transactions on Wireless Communications*, vol. 7, no. 6, pp. 2131–2142, June 2008.

[9] J Jalden, L G Barbero, B Ottersten, and J S Thompson, "The Error Probability of the Fixed-Complexity Sphere Decoder," *Signal Processing, IEEE Transactions on*, vol. 57, no. 7, pp. 2711–2720, July 2009.

[10] D. Wubben, R Böhnke, V Kuhn, and K.-D. Kammeyer, "MMSE Extension of V-BLAST Based on Sorted QR Decomposition," in *Vehicular Technology Conference, 2003. VTC 2003-Fall. 2003 IEEE 58th*, 2003, vol. 1, pp. 508–512 Vol.1.

[11] P Luethi, A Burg, S Haene, D Perels, N Felber, and W Fichtner, "VLSI Implementation of a High-Speed Iterative Sorted MMSE QR Decomposition," in *Circuits and Systems, 2007. ISCAS 2007. IEEE International Symposium on*, May 2007, pp. 1421–1424.

[12] G.L. Nazar, Christina Gimmler, and Norbert Wehn, "Implementation comparisons of the QR decomposition for MIMO detection," in *Proceedings of the 23rd symposium on Integrated circuits and system design*. 2010, vol. 1, pp. 210–214, ACM.

[13] Gene H. Golub and Charles F. Van Loan, *Matrix Computations*, The Johns Hopkins University Press, 3rd editio edition, 1996.

[14] Texas Instruments, "TMS320C6474 Multicore Digital Signal Processor," 2011.